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On the $\mathrm{SL}(2,\mathbb{R})$ symmetry in Yang-Mills theories in the Landau, Curci-Ferrari and maximal abelian gauge

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Abstract: The existence of a $\mathrm{SL}(2,\mathbb{R})$ symmetry is discussed in $\mathrm{SU}(N)$ Yang-Mills in the maximal abelian gauge. This symmetry, also present in the Landau and Curci-Ferrari gauge, ensures the absence of tachyons in the maximal abelian gauge. In all these gauges, $\mathrm{SL}(2,\mathbb{R})$ turns out to be dynamically broken by ghost condensates.

Keywords: BRST Quantization, Nonperturbative Effects, Confinement.

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Contents

1. Introduction

It is widely believed that the dual superconductivity mechanism [1, 2] can be at the origin of color confinement. The key ingredients of this mechanism are the abelian dominance and the monopoles condensation. According to the dual superconductivity picture, the low energy behavior of QCD should be described by an effective abelian theory in the presence of monopoles. The condensation of the monopoles gives rise to the formation of Abrikosov-Nielsen-Olesen flux tubes which confine all chromoelectric charges. This mechanism has received many confirmations from lattice simulations in abelian gauges, which are very useful in order to characterize the effective relevant degrees of freedom at low energies.

Among the abelian gauges, the so called maximal abelian gauge (MAG) plays an important role. This gauge, introduced in [2, 3], has given evidences for the abelian dominance and for the monopoles condensation, while providing a renormalizable gauge in the continuum. Here, the abelian degrees of freedom are identified with the components of the gauge field belonging to the Cartan subgroup of the gauge group $\mathrm{SU}(N)$. The other components correspond to the $(N^2 - N)$ off-diagonal generators of $\mathrm{SU}(N)$ and, being no longer protected by gauge invariance, are expected to acquire a mass, thus decoupling at low energies. The understanding of the mechanism for the dynamical mass generation of the off-diagonal components is fundamental for the abelian dominance.

A feature to be underlined is that the MAG is a nonlinear gauge. As a consequence, a quartic self-interaction term in the Faddeev-Popov ghosts is necessarily required for renormalizability [4, 5]. Furthermore, as discussed in [6, 7] and later on in [8], the four ghost interaction gives rise to an effective potential whose vacuum configuration favors the formation of off-diagonal ghost condensates $\langle c\overline{c}\rangle$, $\langle cc\rangle$ and $\langle \overline{c}\overline{c}\rangle$.[1] However, these ghost condensates were proven [9] to originate an unwanted effective tachyon mass for the off-diagonal gluons, due to the presence in the MAG of an off-diagonal interaction term of the type $AAc\overline{c}$.

Meanwhile, the ghost condensation has been observed in others gauges, namely in the Curci-Ferrari gauge [10] and in the Landau gauge [11]. We remark that in these gauges the ghost condensates do not give rise to any mass term for the gauge fields. The existence of these condensates turns out to be related to the dynamical breaking of a $\mathrm{SL}(2,\mathbb{R})$ symmetry which is known to be present in both Curci-Ferrari and Landau gauge since long time [12]–[15]. It is worth noticing that in the Curci-Ferrari gauge a BRST invariant mass term $(\frac{1}{2}\mathcal{A}^2 - \xi c\overline{c}) \equiv (\frac{1}{2}\mathcal{A}^A\mathcal{A}^A - \xi c^A\overline{c}^A)$, with $A = 1, \ldots, N^2-1$ and ξ being the gauge parameter, can be introduced without spoiling the renormalizability of the model [16, 17].

Recent investigations [18] have suggested that the existence of a nonvanishing condensate $(\frac{1}{2}\langle\mathcal{A}^2\rangle - \xi\langle c\overline{c}\rangle)$ could yield a BRST invariant dynamical mass for both gluons and ghosts. We observe that in the limit $\xi \to 0$, this condensate reduces to the pure gauge condensate $\frac{1}{2}\langle\mathcal{A}^2\rangle$ whose existence is well established in the Landau gauge [19, 20], providing indeed an effective mass for the gluons.

The aim of this paper is to show that the $\mathrm{SL}(2,\mathbb{R})$ symmetry is also present in the MAG for $\mathrm{SU}(N)$ Yang-Mills, with any value of N. This result generalizes that of [6, 7, 21], where the $\mathrm{SL}(2,\mathbb{R})$ symmetry has been established only for a partially gauge-fixed version of the action. In particular, the requirement of an exact $\mathrm{SL}(2,\mathbb{R})$ invariance for the complete quantized action, including the diagonal part of the gauge fixing, will have very welcome consequences. In fact, as we shall see, this requirement introduces new interaction terms in the action, which precisely cancel the term $AAc\overline{c}$ responsible for the generation of the tachyon mass. In other words, no tachyons are present if the $\mathrm{SL}(2,\mathbb{R})$ symmetry is required from the beginning as an exact invariance of the fully gauge-fixed action.

This observation allows us to make an interesting *trait d' union* between the Landau gauge, the Curci-Ferrari gauge and the MAG, providing a more consistent and general understanding of the ghost condensation and of the mechanism for the dynamical generation of the effective gluon masses. The whole framework can be summarized as follows. The ghost condensates signal the dynamical breaking of the $\mathrm{SL}(2,\mathbb{R})$ symmetry, present in all these gauges. Also, the condensed vacuum has the interesting property of leaving unbroken the Cartan subgroup of the gauge group. As a consequence of the ghost condensation, the off-diagonal ghost propagators get deeply modified in the infrared region [6, 7, 8, 11]. This feature might be relevant for the analysis of the infrared behavior of the gluon propagator, through the ghost-gluon mixed Schwinger-Dyson equations. Moreover, the ghost condensates contribute to the dimension four condensate $\left\langle \frac{\alpha}{\pi}F^2 \right\rangle$ through the trace anomaly.

On the other hand, the dynamical mass generation for the gluons is expected to be related to the BRST invariant condensate $(\frac{1}{2}\langle\mathcal{A}^2\rangle - \xi\langle c\overline{c}\rangle)$. It is remarkable that this

[1] The notation for the ghost condensates $\langle c\overline{c}\rangle$, $\langle cc\rangle$ and $\langle \overline{c}\overline{c}\rangle$ stands for $\langle f^{iab}c^a\overline{c}^b\rangle$, $\langle f^{iab}c^ac^b\rangle$ and $\langle f^{iab}\overline{c}^a\overline{c}^b\rangle$, where f^{iab} are the structure constants of the gauge group. The index i runs over the Cartan generators, while the indices a, b correspond to the off-diagonal generators.

condensate can be defined in a BRST invariant way also in the MAG [18, 9], where it can give masses for all off-diagonal fields, thus playing a pivotal role for the abelian dominance.

The paper is organized as follows. Section 2 is devoted to the analysis of the $SL(2,\mathbb{R})$ symmetry in the Landau, Curci-Ferrari and MAG gauge. In section 3 we prove that the requirement of the $SL(2,\mathbb{R})$ symmetry for the MAG in $SU(N)$ Yang-Mills yields a renormalizable theory. In section 4 we discuss the issue of the ghost condensation and of the absence of tachyons. In section 5 we present the conclusions.

2. Yang-Mills theories and the $SL(2,\mathbb{R})$ symmetry

Let $\mathcal{A}_\mu$ be the Lie algebra valued connection for the gauge group $SU(N)$, whose generators $T^A, [T^A, T^B] = f^{ABC}T^C$, are chosen to be antihermitean and to obey the orthonormality condition $\mathrm{Tr}(T^AT^B) = \delta^{AB}$, with $A, B, C = 1, \ldots, (N^2-1)$. The covariant derivative is given by

$$\mathcal{D}_\mu^{AB} \equiv \partial_\mu \delta^{AB} - gf^{ABC}\mathcal{A}_\mu^C \, . \tag{2.1}$$

Let s and $\overline{s}$ be the nilpotent BRST and anti-BRST transformations respectively, acting on the fields as

$$\begin{aligned} s\mathcal{A}_\mu^A &= -\mathcal{D}_\mu^{AB}c^B \\ sc^A &= \frac{g}{2}f^{ABC}c^Bc^C \\ s\overline{c}^A &= b^A \\ sb^A &= 0 \, , \end{aligned} \tag{2.2}$$

$$\begin{aligned} \overline{s}\mathcal{A}_\mu^A &= -\mathcal{D}_\mu^{AB}\overline{c}^B \\ \overline{s}c^A &= -b^A + gf^{ABC}c^B\overline{c}^C \\ \overline{s}\overline{c}^A &= \frac{g}{2}f^{ABC}\overline{c}^B\overline{c}^C \\ \overline{s}b^A &= -gf^{ABC}b^B\overline{c}^C \, . \end{aligned} \tag{2.3}$$

Here c^A and $\overline{c}^A$ generally denote the Faddeev-Popov ghosts and anti-ghosts, while b^A denote the Lagrange multipliers.

Furthermore, we define the operators δ and $\overline{\delta}$ by

$$\begin{aligned} \delta\overline{c}^A &= c^A \\ \delta b^A &= \frac{g}{2}f^{ABC}c^Bc^C \\ \delta\mathcal{A}_\mu^A &= \delta c^A = 0 \, , \end{aligned} \tag{2.4}$$

$$\begin{aligned} \overline{\delta}c^A &= \overline{c}^A \\ \overline{\delta}b^A &= \frac{g}{2}f^{ABC}\overline{c}^B\overline{c}^C \\ \overline{\delta}\mathcal{A}_\mu^A &= \overline{\delta}\overline{c}^A = 0 \, . \end{aligned} \tag{2.5}$$

Together with the Faddeev-Popov ghost number operator δ_{FP}, δ and $\overline{\delta}$ generate a $\mathrm{SL}(2,\mathbb{R})$ algebra. This algebra is a subalgebra of the algebra generated by δ_{FP}, δ, $\overline{\delta}$ and the BRST and anti-BRST operators s and $\overline{s}$. The algebra

$$\begin{aligned}
s^2 &= 0\,, & \overline{s}^2 &= 0\,, \\
\{s,\overline{s}\} &= 0\,, & [\delta,\overline{\delta}] &= \delta_{\mathrm{FP}}\,, \\
[\delta,\delta_{\mathrm{FP}}] &= -2\delta\,, & [\overline{\delta},\delta_{\mathrm{FP}}] &= 2\overline{\delta}\,, \\
[s,\delta_{\mathrm{FP}}] &= -s\,, & [\overline{s},\delta_{\mathrm{FP}}] &= \overline{s}\,, \\
[s,\delta] &= 0\,, & [\overline{s},\overline{\delta}] &= 0\,, \\
[s,\overline{\delta}] &= -\overline{s}\,, & [\overline{s},\delta] &= -s\,,
\end{aligned} \tag{2.6}$$

is known as the Nakanishi-Ojima (NO) algebra [12].

2.1 Landau gauge

In the Landau gauge, we have

$$S = S_{YM} + S_{GF+FP} = -\frac{1}{4}\int d^4x \mathcal{F}^A_{\mu\nu}\mathcal{F}^{A\mu\nu} + s\int d^4x \overline{c}^A \partial_\mu \mathcal{A}^{A\mu}\,. \tag{2.7}$$

The BRST invariance is immediate, just as the δ invariance since

$$\delta S_{GF+FP} = s\int d^4x c^A \partial_\mu \mathcal{A}^{A\mu} = \frac{s^2}{2}\int d^4x \mathcal{A}^2 = 0\,. \tag{2.8}$$

It is easy checked that also $\overline{s}$ and $\overline{\delta}$ leave the action (2.7) invariant. Hence, the NO algebra is a global symmetry of Yang-Mills theories in the Landau gauge, a fact already longer known [12, 13].

2.2 Curci-Ferrari gauge

Next, we discuss Yang-Mills theories in a class of generalized covariant non-linear gauges proposed in [14]. The action is given by

$$\begin{aligned}
S &= S_{YM} + S_{GF+FP} \\
&= -\frac{1}{4}\int d^4x \mathcal{F}^A_{\mu\nu}\mathcal{F}^{A\mu\nu} + s\overline{s}\int d^4x \left(\frac{1}{2}\mathcal{A}^A_\mu\mathcal{A}^{A\mu} - \frac{\xi}{2}c^A\overline{c}^A\right) \\
&= -\frac{1}{4}\int d^4x \mathcal{F}^A_{\mu\nu}\mathcal{F}^{A\mu\nu} + \int d^4x \Big(b^A\partial_\mu\mathcal{A}^{A\mu} + \frac{\xi}{2}b^Ab^A + \overline{c}^A\partial^\mu\mathcal{D}^{AB}_\mu c^B - \frac{\xi}{2}gf^{ABC}b^A\overline{c}^Bc^C - \\
&\qquad - \frac{\xi}{8}g^2 f^{ABC}f^{CDE}\overline{c}^A\overline{c}^Bc^Dc^E\Big),
\end{aligned} \tag{2.9}$$

where ξ is the gauge parameter. A gauge fixing as in (2.9) is sometimes called the Curci-Ferrari (CF) gauge, since its gauge fixing part resembles the gauge fixing part of the massive, $\mathrm{SU}(N)$ gauge model introduced in [16].

In addition to the BRST and anti-BRST symmetries, the action (2.9) is also invariant under the global $\mathrm{SL}(2,\mathbb{R})$ symmetry generated by δ, $\overline{\delta}$ [21, 17] and δ_{FP}. We conclude that Yang-Mills theories in the CF gauge have the NO symmetry.

2.3 Maximal abelian gauge

We decompose the gauge field into its off-diagonal and diagonal parts, namely

$$\mathcal{A}_\mu = \mathcal{A}^A_\mu T^A = A^a_\mu T^a + A^i_\mu T^i \,, \tag{2.10}$$

where the index i labels the $N-1$ generators T^i of the Cartan subalgebra. The remaining $N(N-1)$ off-diagonal generators T^a will be labelled by the index a. Accordingly, the field strength decomposes as

$$\mathcal{F}_{\mu\nu} = \mathcal{F}^A_{\mu\nu} T^A = F^a_{\mu\nu} T^a + F^i_{\mu\nu} T^i \,, \tag{2.11}$$

with the off-diagonal and diagonal parts given respectively by

$$\begin{aligned} F^a_{\mu\nu} &= D^{ab}_\mu A^b_\nu - D^{ab}_\nu A^b_\mu \; + g\, f^{abc} A^b_\mu A^c_\nu \,, \\ F^i_{\mu\nu} &= \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g f^{abi} A^a_\mu A^b_\nu \,, \end{aligned} \tag{2.12}$$

where the covariant derivative D^{ab}_μ is defined with respect to the diagonal components A^i_μ

$$D^{ab}_\mu \equiv \partial_\mu \delta^{ab} - g f^{abi} A^i_\mu \,. \tag{2.13}$$

For the pure Yang-Mills action one obtains

$$S_{\rm YM} = -\frac{1}{4} \int d^4x \, \left(F^a_{\mu\nu} F^{a\mu\nu} + F^i_{\mu\nu} F^{i\mu\nu} \right) . \tag{2.14}$$

The so called MAG gauge condition amounts to fix the value of the covariant derivative $(D^{ab}_\mu A^{b\mu})$ of the off-diagonal components [2, 3]. However, this condition being nonlinear, a quartic ghost self-interaction term is required for renormalizability [4, 5]. The corresponding gauge fixing term turns out to be [22]

$$S_{\rm MAG} = s\overline{s} \int d^4x \, \left(\frac{1}{2} A^a_\mu A^{a\mu} - \frac{\xi}{2} c^a \overline{c}^a \right) \,, \tag{2.15}$$

where s denotes the nilpotent BRST operator

$$\begin{aligned} sA^a_\mu &= -\left(D^{ab}_\mu c^b + g f^{abc} A^b_\mu c^c + g f^{abi} A^b_\mu c^i \right) \,, \qquad sA^i_\mu = -\left(\partial_\mu c^i + g f^{iab} A^a_\mu c^b \right) \,, \\ sc^a &= g f^{abi} c^b c^i + \frac{g}{2} f^{abc} c^b c^c \,, \qquad sc^i = \frac{g}{2} f^{iab} c^a c^b, \\ s\overline{c}^a &= b^a \,, \qquad s\overline{c}^i = b^i \,, \\ sb^a &= 0 \,, \qquad sb^i = 0 \,, \end{aligned} \tag{2.16}$$

and $\overline{s}$ the nilpotent anti-BRST operator, which acts as

$$\begin{aligned} \overline{s}A^a_\mu &= -\left(D^{ab}_\mu \overline{c}^b + g f^{abc} A^b_\mu \overline{c}^c + g f^{abi} A^b_\mu \overline{c}^i \right), \qquad \overline{s}A^i_\mu = -\left(\partial_\mu \overline{c}^i + g f^{iab} A^a_\mu \overline{c}^b \right), \\ \overline{s}\overline{c}^a &= g f^{abi} \overline{c}^b \overline{c}^i + \frac{g}{2} f^{abc} \overline{c}^b \overline{c}^c \,, \qquad \overline{s}\overline{c}^i = \frac{g}{2} f^{iab} \overline{c}^a \overline{c}^b \,, \\ \overline{s}c^i &= -b^i + g f^{ibc} c^b \overline{c}^c \,, \qquad \overline{s}b^i = -g f^{ibc} b^b \overline{c}^c \,, \\ \overline{s}c^a &= -b^a + g f^{abc} c^b \overline{c}^c + g f^{abi} c^b \overline{c}^i + g f^{abi} \overline{c}^b c^i \,, \\ \overline{s}b^a &= -g f^{abc} b^b \overline{c}^c - g f^{abi} b^b \overline{c}^i + g f^{abi} \overline{c}^b b^i \,. \end{aligned} \tag{2.17}$$

Here c^a and c^i are the off-diagonal and the diagonal components of the Faddeev-Popov ghost field, $\overline{c}^a$ and $\overline{c}^i$ the off-diagonal and the diagonal anti-ghost fields and b^a and b^i are the off-diagonal and diagonal Lagrange multipliers. These transformations are nothing else than the projection on diagonal and off-diagonal fields of (2.2) and (2.3). Expression (2.15) is easily worked out and yields

$$\begin{aligned}
S_{\mathrm{MAG}} &= s\int d^4x\,\left(\overline{c}^a\left(D^{ab}_\mu A^{b\mu}+\frac{\xi}{2}b^a\right)-\frac{\xi}{2}gf^{\,abi}\overline{c}^a\overline{c}^b c^i-\frac{\xi}{4}gf^{\,abc}c^a\overline{c}^b\overline{c}^c\right)\\
&= \int d^4x\Bigg(b^a\left(D^{ab}_\mu A^{b\mu}+\frac{\xi}{2}b^a\right)+\overline{c}^a D^{ab}_\mu D^{\mu bc}c^c+g\overline{c}^a f^{abi}\left(D^{bc}_\mu A^{c\mu}\right)c^i+\\
&\qquad +g\overline{c}^a D^{ab}_\mu\left(f^{bcd}A^{c\mu}c^d\right)-g^2 f^{abi}f^{cdi}\overline{c}^a c^d A^b_\mu A^{c\mu}-\xi g f^{abi}b^a\overline{c}^b c^i-\\
&\qquad -\frac{\xi}{2}gf^{abc}b^a\overline{c}^b c^c-\frac{\xi}{4}g^2 f^{abi}f^{cdi}\overline{c}^a\overline{c}^b c^c c^d-\frac{\xi}{4}g^2 f^{abc}f^{adi}\overline{c}^b\overline{c}^c c^d c^i-\\
&\qquad -\frac{\xi}{8}g^2 f^{abc}f^{ade}\overline{c}^b\overline{c}^c c^d c^e\Bigg)\,. \qquad (2.18)
\end{aligned}$$

The MAG condition allows for a residual local $\mathrm{U}(1)^{N-1}$ invariance with respect to the diagonal subgroup, which has to be fixed by means of a suitable further gauge condition on the diagonal components A^i_μ. We shall choose a diagonal gauge fixing term which is BRST and anti-BRST invariant. The diagonal gauge fixing is then given by

$$\begin{aligned}
S_{\mathrm{diag}} &= s\overline{s}\int d^4x\,\left(\frac{1}{2}A^i_\mu A^{i\mu}\right)\\
&= s\int d^4x\,\left(\overline{c}^i\partial_\mu A^{i\mu}-gf^{iab}A^i_\mu A^{a\mu}\overline{c}^b\right)\\
&= \int d^4x\Bigg(b^i\partial_\mu A^{i\mu}+\overline{c}^i\partial^2 c^i+gf^{iab}A^a_\mu\left(\partial_\mu c^i\overline{c}^b-\partial_\mu\overline{c}^i c^b\right)+\\
&\qquad +g^2 f^{iab}f^{icd}\overline{c}^a c^d A^b_\mu A^{c\mu}-gf^{iab}A^i_\mu A^{a\mu}\left(b^b-gf^{ibc}\overline{c}^c c^i\right)+\\
&\qquad +gf^{iab}A^{i\mu}\left(D^{ac}_\mu c^c\right)\overline{c}^b+g^2 f^{abi}f^{acd}A^i_\mu A^{c\mu}c^d\overline{c}^b\Bigg)\,. \qquad (2.19)
\end{aligned}$$

In addition to the BRST and the anti-BRST symmetry, the gauge-fixed action ($S_{\mathrm{YM}}+S_{\mathrm{MAG}}+S_{\mathrm{diag}}$) is invariant under a global $\mathrm{SL}(2,\mathbb{R})$ symmetry, which is generated by the operators δ, $\overline{\delta}$ and the ghost number operator δ_{FP}. For the δ transformations we have

$$\begin{aligned}
\delta\overline{c}^a &= c^a\,, \qquad \delta\overline{c}^i = c^i\,,\\
\delta b^a &= gf^{abi}c^b c^i+\frac{g}{2}f^{abc}c^b c^c\,,\\
\delta b^i &= \frac{g}{2}f^{iab}c^a c^b\,,\\
\delta A^a_\mu &= \delta A^i_\mu = \delta c^a = \delta c^i = 0\,. \qquad (2.20)
\end{aligned}$$

The operator $\overline{\delta}$ acts as

$$\begin{aligned}
\overline{\delta}c^a &= \overline{c}^a\,, \qquad \overline{\delta}c^i = \overline{c}^i\,,\\
\overline{\delta}b^a &= gf^{abi}\overline{c}^b\overline{c}^i+\frac{g}{2}f^{abc}\overline{c}^b\overline{c}^c\,,
\end{aligned}$$

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$$\overline{\delta} b^i = \frac{g}{2} f^{iab} \overline{c}^a \overline{c}^b \,,$$
$$\overline{\delta} A^a_\mu = \overline{\delta} A^i_\mu = \overline{\delta} \overline{c}^a = \overline{\delta} \overline{c}^i = 0 \,. \tag{2.21}$$

The existence of the SL$(2, \mathbb{R})$symmetry has been pointed out in [6] in the maximal abelian gauge for the gauge group SU(2). A generalization of it can be found in [21]. There are, however, important differences between [6, 21] and the present analysis.

The first point relies on the choice of the diagonal part of the gauge fixing $S_{\rm diag}$, a necessary step towards a complete quantization of the model. We remark that with our choice of $S_{\rm diag}$ in eq. (2.19), the whole NO algebra becomes an exact symmetry of the gauge fixed action $(S_{\rm YM} + S_{\rm MAG} + S_{\rm diag})$ with gauge group SU(N), for any value of N. In particular, as one can see from expression (2.19), $S_{\rm diag}$ contains the interaction term $g^2 f^{iab} f^{icd} \overline{c}^a c^d A^b_\mu A^{c\mu}$, which precisely cancels the corresponding term appearing in eq.(2.18) for $S_{\rm MAG}$. This is a welcome feature, implying that no tachyons are generated if the SL$(2, \mathbb{R})$, and thus the NO algebra, is required as an exact invariance for the starting gauge-fixed action. We remark that a similar compensation holds also for the interaction terms of (2.19) and (2.18) containing two diagonal gluons and a pair of off-diagonal ghost-antighost, implying that the diagonal gauge fields remain massless.

A second difference concerns the way the fields are transformed. We observe that in the present case, the field transformations (2.16)–(2.17) and (2.20)–(2.21) are obtained from (2.2)–(2.3) and (2.4)–(2.5) upon projection of the group index $A = 1, \ldots, \left(N^2 - 1\right)$ over the Cartan subgroup of SU(N) and over the off-diagonal generators, thus preserving the whole NO structure. As it is apparent from eqs. (2.20), (2.21), the diagonal fields c^i, $\overline{c}^i$, b^i transform nontrivially, a necessary feature for the NO algebra. These transformations were not taken into account in the original work [6]. Also, in [21], the NO structure is analysed only on the off-diagonal fields, the diagonal components $\overline{c}^i$, b^i being set to zero.

In summary, it is possible to choose the diagonal part $S_{\rm diag}$ of the gauge fixing so that the whole NO structure is preserved. Remarkably, the requirement that the NO algebra is an exact symmetry of the starting action ensures that no tachyons show up. It remains now to prove that the choice of the diagonal gauge fixing (2.19) will lead to a renormalizable model. This will be the task of the next section.

3. Stability of the MAG under radiative corrections

In order to discuss the renormalizability of the action $(S_{\rm YM} + S_{\rm MAG} + S_{\rm diag})$ within the BRST framework, we have to write down the Ward identities corresponding to the symmetries of the classical action. The expression of the BRST invariance as a functional identity requires the introduction of invariant external sources

$$S_{\rm ext} = \int d^4x \left(\Omega^{a\mu} s A^a_\mu + \Omega^{i\mu} s A^i_\mu + L^a s c^a + L^i s c^i \right) , \tag{3.1}$$

with $s\Omega^a_\mu = s\Omega^i_\mu = sL^a = sL^i = 0$.

The δ transformations of the external fields can be fixed by imposing $\delta S_{\rm ext} = 0$, which yields $\delta\Omega^a_\mu = \delta\Omega^i_\mu = \delta L^a = \delta L^i = 0$. Therefore the classical action

$$\Sigma = S_{\rm YM} + S_{\rm MAG} + S_{\rm diag} + S_{\rm ext} \,, \tag{3.2}$$

is invariant under BRST and δ transformations, obeying the following identities

- Slavnov-Taylor identity:

$$\mathcal{S}(\Sigma) = \int d^4x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta\Sigma}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta A^i_\mu}\frac{\delta\Sigma}{\delta\Omega^{i\mu}} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^i}\frac{\delta\Sigma}{\delta L^i} + b^a\frac{\delta\Sigma}{\delta\overline{c}^a} + b^i\frac{\delta\Sigma}{\delta\overline{c}^i} \right) = 0 \tag{3.3}$$

- δ symmetry Ward identity:

$$\mathcal{D}(\Sigma) = \int d^4x \left(c^a\frac{\delta\Sigma}{\delta\overline{c}^a} + c^i\frac{\delta\Sigma}{\delta\overline{c}^i} + \frac{\delta\Sigma}{\delta b^a}\frac{\delta\Sigma}{\delta L^a} + \frac{\delta\Sigma}{\delta b^i}\frac{\delta\Sigma}{\delta L^i} \right) = 0 \tag{3.4}$$

- Integrated diagonal ghost equation:

$$\mathcal{G}^i\Sigma = \Delta^i_{\rm cl} \,, \tag{3.5}$$

  where

$$\mathcal{G}^i = \int d^4x \left(\frac{\delta}{\delta c^i} + gf^{\,abi}\overline{c}^a\frac{\delta}{\delta b^b} \right) \tag{3.6}$$

  and the classical breaking $\Delta^i_{\rm cl}$ is given by

$$\Delta^i_{\rm cl} = \int d^4x \left(gf^{\,abi}\Omega^a_\mu A^{b\mu} - gf^{\,abi}L^a c^b \right) . \tag{3.7}$$

- Integrated diagonal antighost equation:

$$\int d^4x \frac{\delta\Sigma}{\delta\overline{c}^i} = 0 \,. \tag{3.8}$$

Similarly, we could also impose the anti-BRST and $\overline{\delta}$ symmetries in a functional way by introducing an additional set of external sources. However, the Ward identities (3.3)–(3.8) are sufficient to ensure the stability of the classical action under quantum corrections. It is not difficult indeed, by using the algebraic renormalization procedure [23, 24], to prove that the model is renormalizable.

4. Ghost condensation and the breakdown of $\mathrm{SL}(2,\mathbb{R})$ and NO symmetry

In this section, we give a brief discussion of the existence of ghost condensates and of their relationship with $\mathrm{SL}(2,\mathbb{R})$ and hence with NO symmetry.

The condensation of ghosts came to attention originally in the works of [6, 7] in the context of SU(2) MAG. The decomposition of the 4-ghost interaction allowed to construct an effective potential with a nontrivial minimum for the off-diagonal condensate $\langle \varepsilon^{3ab}c^a\overline{c}^b \rangle$. This condensate implies a dynamical breaking of $\mathrm{SL}(2,\mathbb{R})$. Order parameters for this breaking are given by

$$\left\langle \varepsilon^{3ab}c^a\overline{c}^b \right\rangle = \frac{1}{2}\left\langle \delta\left(\varepsilon^{3ab}\overline{c}^a\overline{c}^b \right) \right\rangle = \frac{1}{2}\left\langle \overline{\delta}\left(\varepsilon^{3ab}c^a c^b \right) \right\rangle . \tag{4.1}$$

As a consequence the generators δ and $\overline{\delta}$ of $\mathrm{SL}(2,\mathbb{R})$ are broken [6], while the Faddeev-Popov ghost number generator $\delta_{FP} = \left[\delta, \overline{\delta}\right]$ remains unbroken. The ghost condensate resulted in a mass for the off-diagonal gluons, whose origin can be traced back to the presence of the off-diagonal interaction term $\overline{c}cA_\mu A^\mu$ term in expression (2.18). However, as was shown in [9], this mass is a tachyonic one. Furthermore, the requirement of invariance of S_{diag} under anti-BRST and, as a consequence, under $\mathrm{SL}(2,\mathbb{R})$ transformations, also gives rise to quartic terms of the kind $\overline{c}cA_\mu A^\mu$ (see (2.19)), which precisely cancel those of S_{MAG} in (2.18). Thus, if the $\mathrm{SL}(2,\mathbb{R})$ symmetry (NO symmetry (2.6)) is required, the ghost condensates do not induce any unphysical mass for the off-diagonal gluons.

The 4-ghost interaction can be decomposed in a different way, so that the ghost condensation takes places in the Faddeev-Popov charged channels $\varepsilon^{3ab}c^a c^b$ and $\varepsilon^{3ab}\overline{c}^a\overline{c}^b$ instead of $\varepsilon^{3ab}c^a\overline{c}^b$. In this case, the ghost number symmetry is broken [8]. Consequently, the NO algebra is broken again. It is interesting to see that the existence of different ghost channels in which the ghost condensation can take place has an analogy in ordinary superconductivity, known as BCS (particle-particle and hole-hole pairing) versus Overhauser (particle-hole paring) [25, 26] . In the present case the BCS channel corresponds to the Faddeev-Popov charged condensates $\left\langle\varepsilon^{3ab}c^a c^b\right\rangle$ and $\left\langle\varepsilon^{3ab}\overline{c}^a\overline{c}^b\right\rangle$, while the Overhauser channel to $\left\langle\varepsilon^{3ab}c^a\overline{c}^b\right\rangle$.

The CF gauge (2.9) contains a 4-ghost interaction too, so it is expected that the ghost condensation can take place also in the CF gauge. This was confirmed in [10]. Notice that the CF and MAG gauges look very similar, and since the CF gauge does not contain terms like $\overline{c}c\mathcal{A}_\mu\mathcal{A}^\mu$, no (tachyonic) mass is induced for the gluons. This can be seen as some extra evidence why also in the MAG no tachyon mass terms should be generated.

More surprising is the fact that also in the Landau gauge, the ghost condensation occurs [11]. Since there is no 4-ghost interaction to be decomposed, another technique was used to discuss this gauge. A combination of the algebraic renormalization technique [23, 24] and the local composite operator technique [19] allowed a clean treatment, with the result that also in case of the Landau gauge the NO symmetry is broken.

Since the aforementioned ghost condensates are not giving masses for the gluons, one could wonder what the mechanism behind the dynamical generation of gluon masses could be. It was proposed in [18] that the generation of a real mass for the gluons in case of the CF gauge is related to a non-vanishing vacuum expectation value for the two-dimensional local, composite operator $\left(\frac{1}{2}\mathcal{A}_\mu^A\mathcal{A}^{A\mu} - \xi c^A\overline{c}^A\right)$. It is interesting to notice that this is exactly the kind of mass term that is present in the massive lagrangian of Curci and Ferrari [16]. In the case of MAG, the relevant operator is believed to be $\left(\frac{1}{2}A_\mu^a A^{a\mu} - \xi c^a\overline{c}^a\right)$, and is expected to provide an effective mass for both off-diagonal gauge and off-diagonal ghost fields [18, 9].

5. Conclusion

- In this paper the presence of the $\mathrm{SL}(2,\mathbb{R})$ symmetry has been analysed in the Landau, Curci-Ferrari and maximal abelian gauge for $\mathrm{SU}(N)$ Yang-Mills. In all these gauges $\mathrm{SL}(2,\mathbb{R})$ can be established as an exact invariance of the complete gauge fixed action. Together with the BRST and anti-BRST, the generators of $\mathrm{SL}(2,\mathbb{R})$ are part of a larger algebra, known as the Nakanishi-Ojima algebra [12].

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- In particular, we have been able to show that in the case of the maximal abelian gauge, the requirement of $SL(2,\mathbb{R})$ for the complete action, including the diagonal gauge fixing term, ensures that no tachyons will be generated.

- In all these gauges, the $SL(2,\mathbb{R})$ symmetry turns out to be dynamically broken by the existence of off-diagonal ghost condensates $\langle cc \rangle$, $\langle \overline{c}\overline{c} \rangle$ and $\langle c\overline{c} \rangle$. As a consequence, the NO algebra is also broken. These condensates deeply modify the infrared behavior of the off-diagonal ghost propagator, while contributing to the vacuum energy density and hence to the trace anomaly [6, 7, 8].

- Finally, let us spend a few words on future research. As already remarked, the ghost condensation can be observed in different channels, providing a close analogy with the BCS versus Overhauser effect of superconductivity. We have also pointed out that the existence of the condensate $\left(\frac{1}{2}\left\langle A^a_\mu A^{a\mu}\right\rangle - \xi\left\langle c^a\overline{c}^a\right\rangle\right)$ can be at the origin of the dynamical mass generation in the MAG for all off-diagonal gluons and ghosts [18, 9], a feature of great relevance for the abelian dominance. Both aspects will be analysed by combining the algebraic renormalization [23, 24] with the local composite operator technique [19], as done in the case of the ghost condensation in the Landau gauge [11]. The combination of these two procedures results in a very powerful framework for discussing the ghost condensation in the various channels as well as for studying the condensate $\left(\frac{1}{2}\left\langle A^a_\mu A^{a\mu}\right\rangle - \xi\left\langle c^a\overline{c}^a\right\rangle\right)$ and its relationship with the dynamical mass generation. Also, the detailed analysis of the decoupling at low energies of the diagonal ghosts and of the validity of the local $U(1)^{N-1}$ Ward identity in the MAG deserves careful attention.

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